The Propagation of Signals Along a Three-Layered Region: Microstrip

RONOLD W. P. KING, LIFE FELLOW, IEEE

Abstract —The tangential components of the electric field on the air–substrate surface of a three-layered region such as microstrip are determined when the source is a unit electric dipole on that surface. This is done by integrating the rigorous Hankel transforms subject to the condition $k_0^2 < |k_1^2| \ll |k_2^2|$, where k_0 is the wavenumber of air, k_1 of the substrate, and k_2 of the conductor. It is found that the field consists of lateral-wave terms and direct-wave terms with different wavenumbers and phase velocities. The significance of these characteristics is discussed with reference to dispersion and coupling.

I. Introduction

An IMPORTANT type of electric circuit, illustrated in Fig. 1, consists of conducting paths in the form of thin copper strips on a dielectric layer or substrate that is bounded on one side by air, on the other side by a conducting sheet that is thick compared to the skin depth. The substrate is bonded to the conducting sheet and the combination is generally known as microstrip. Microstrip circuits, including especially transmission lines and antennas, are printed or etched on the substrate and, at appropriate points, connected through it to the conducting sheet. They are of primary importance in the frequency band from 0.1 to 100 GHz.

Microstrip transmission lines have been a subject of theoretical and experimental investigation for more than a quarter century. It is well known that they do not support a TEM mode [1], [2], as do the familiar coaxial and open-wire lines; even TE and TM modes of conventional type are not possible. As a consequence, the accurate analysis of microstrip transmission lines is difficult. A principal approach was introduced by Wu [1] in the form of a Fourier transform. A generalization of this method to include the application of various integral transforms, such as the Fourier and Hankel transforms, to boundary-value problems such as microstrip is commonly referred to as the spectral-domain approach [3]. There is a very extensive literature on this and related subjects [4]–[9]. Unfortunately, the application of an integral transform and the subsequent solution of the closed-form integral equation by numerical methods is a complicated and time-consuming process, not conveniently used with practical circuits. Alternative approximate procedures include (1) the determination of the effective permittivity of an "equivalent" single-dielectric, TEM-mode transmission line with a phase velocity equal to that of the microstrip line [10], and (2) the assumption that "the quasi-TEM mode on microstrip is primarily a single, longitudinal-section electric mode" combined with the introduction of an appropriate model with a U-shaped cross section that can be analyzed and that has dispersion characteristics resembling those of microstrip [11]. These considerations are no less significant for transient signals and pulses [12]. Similar problems of dispersion and coupling occur in the analysis of microstrip antennas [13], [14].

It is, of course, well known that a dielectric-coated conducting plane supports a surface wave that propagates in both the dielectric and the air [15]. It is derived as a possible solution of Maxwell's equations when the appropriate conditions are imposed. These include the requirement that the electromagnetic field in the air be exponentially attenuated in the direction normal to the surface of the dielectric. The phase velocity of propagation along the surface in both air and the dielectric is intermediate to the velocity c in air and $c\epsilon^{-1/2}$ in the dielectric. Although a possible solution of Maxwell's equations, this type of surface wave is not excited by currents in conductors on the air–substrate interface.

The rigorous approach to the analysis of microstrip circuits and the resolution of perplexing problems of dispersion and coupling must necessarily follow the path already taken in the spectral-domain formulation. However, there is an alternative to attempting to solve the resulting complicated and physically obscure integrals numerically. It is to seek a fundamental understanding of the nature of electromagnetic wave propagation in microstrip by obtaining simple analytical approximations of the integrals in a manner similar to that carried out for the problem of two electrically different half-spaces [16].

Actually microstrip is a three-layered region the properties of which can be investigated in terms of the electromagnetic field generated by a horizontal electric dipole on the air–dielectric surface—which can be looked upon as an element of a transmission line or antenna—and vertical electric dipoles in the dielectric—which correspond to elements of a vertical conductor joining a horizontal circuit through the substrate to the conducting plane.

Manuscript received November 7, 1987; revised January 28, 1988. This work was supported in part by the Joint Services Electronics Program under Contract N00014-84-K-0465 with Harvard University and in part by the U.S. Army Research Office under Contract DAAG29-84-K-0188 with Tufts University.

The author is with the Gordon McKay Laboratory, Harvard University, Cambridge, MA 02138.

IEEE Log Number 8821065.

0018-9480/88/0600-1080$01.00

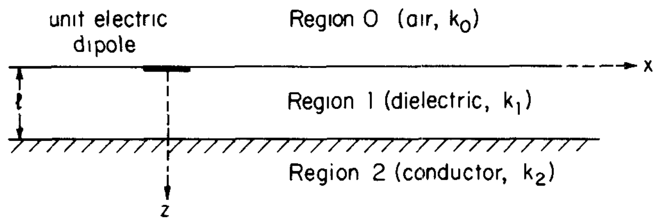

Fig. 1. Unit electric dipole on plane boundary $z=0$ between air and a sheet of dielectric with thickness l over a conducting ground plane.

II. The Field of a Horizontal Electric Dipole on the Air–Dielectric Boundary

The circuit to be treated is illustrated in Fig. 1. It consists of a unit electric dipole (electric moment $I_x(0)h_e = 1$ Am) on the air–dielectric boundary with its axis parallel to the x axis and with its center at the origin, $x=y=z=0$. The air is region 0 ($z \leqslant 0$, wavenumber $k_0 = \omega/c$); the dielectric is region 1 ($0 \leqslant z \leqslant l$, wavenumber $k_1 = \beta_1 + i\alpha_1$, $\alpha_1 \ll \beta_1$); and the conducting half-space is region 2 ($l \leqslant z \leqslant \infty$, wavenumber $k_2 = \beta_2 + i\alpha_2$, $\alpha_2 \sim \beta_2$). In typical applications, $l \sim 0.1$ cm, $\epsilon_{1r} \sim 2$–4, $f \sim 0.8$ GHz, and $\sigma_3 \sim 1.7\times 10^6$ S/m, or $l \sim 0.66$ cm, $\epsilon_{1r} > 9$, $f \sim 0.5$–7 GHz. In millimeter-wave applications, f can be as high as 94 GHz.

The determination of the electromagnetic field in relatively simple form is made possible when the following inequalities are satisfied:

$$k_0^2 < |k_1^2| \ll |k_2^2|. \tag{1}$$

For the tangential components E_ρ and E_ϕ, primary interest is in the field at or near the air–dielectric boundary, i.e., at or near $z=0$; for the vertical component E_z, interest is in the field in the dielectric, i.e., $0 \leqslant z < l$. The source dipole is located on the boundary at $d=0$. Because the tangential components of the electric field are continuous across the boundary, it follows that in the cylindrical coordinates ρ, ϕ, z, $E_{0\rho}(\rho,\phi,0) = E_{1\rho}(\rho,\phi,0)$; $E_{0\phi}(\rho,\phi,0) = E_{1\phi}(\rho,\phi,0)$. However, $k_0^2 E_{0z}(\rho,\phi,0) = k_1^2 E_{1z}(\rho,\phi,0)$.

These components are given by the following integrals:

$$\left.\begin{array}{l} E_{1\rho}(\rho,\phi,0) \\ E_{1\phi}(\rho,\phi,0) \end{array}\right\} = \mp \frac{\omega\mu_0}{4\pi k_1^2}\left\{\begin{array}{l}\cos\phi \\ \sin\phi\end{array}\right\} \times \left(\int_0^\infty 2\left\{\frac{\gamma_1}{2}[J_0(\lambda\rho) \mp J_2(\lambda\rho)] + \frac{k_1^2}{2\gamma_1}[J_0(\lambda\rho) \pm J_2(\lambda\rho)]\right\}\lambda\, d\lambda + \int_0^\infty \left\{\frac{\gamma_1}{2}(Q_3-1)[J_0(\lambda\rho) \mp J_2(\lambda\rho)] - \frac{k_1^2}{2\gamma_1}(P_3+1)[J_0(\lambda\rho) \pm J_2(\lambda\rho)]\right\}\lambda\, d\lambda\right) \tag{2}$$

$$E_{1z}(\rho,\phi,z) = \frac{i\omega\mu_0}{4\pi k_1^2}\cos\phi \int_0^\infty (Q_3+1) J_1(\lambda\rho) e^{i\gamma_1 z}\lambda^2\, d\lambda \tag{3}$$

where $J_0(\lambda\rho)$, $J_1(\lambda\rho)$, and $J_2(\lambda\rho)$ are Bessel functions,

$$\gamma_j = \left(k_j^2 - \lambda^2\right)^{1/2} \tag{4}$$

and, for the three-layered region,

$$-f_{er} = Q_3 = \frac{k_2^2\gamma_0 - k_0^2\gamma_2 - i\left[k_1^2\gamma_0(\gamma_2/\gamma_1) - k_0^2\gamma_1\left(k_2^2/k_1^2\right)\right]\tan\gamma_1 l}{k_2^2\gamma_0 + k_0^2\gamma_2 - i\left[k_1^2\gamma_0(\gamma_2/\gamma_1) + k_0^2\gamma_1\left(k_2^2/k_1^2\right)\right]\tan\gamma_1 l} \tag{5}$$

$$-f_{mr} = P_3 = \frac{\gamma_0 - \gamma_2 - i[\gamma_0(\gamma_2/\gamma_1) - \gamma_1]\tan\gamma_1 l}{\gamma_0 + \gamma_2 - i[\gamma_0(\gamma_2/\gamma_1) + \gamma_1]\tan\gamma_1 l}. \tag{6}$$

Here f_{er} and f_{mr} are the reflection coefficients at the air–dielectric boundary for the three-layered region. When the thickness l of the layer becomes infinite so that $\tan\gamma_1 l \to i$ and only the two half-spaces—region 0 and region 1—remain,

$$-f_{er} = Q = \frac{k_1^2\gamma_0 - k_0^2\gamma_1}{k_1^2\gamma_0 + k_0^2\gamma_1} \qquad -f_{mr} = P = \frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1}. \tag{7}$$

These are the negative of the reflection coefficients for the two-layered region. Similarly, when $l \to 0$, $\tan\gamma_1 l \to 0$ and

$$-f_{er} = Q = \frac{k_2^2\gamma_0 - k_0^2\gamma_2}{k_2^2\gamma_0 + k_0^2\gamma_2} \qquad -f_{mr} = P = \frac{\gamma_0 - \gamma_2}{\gamma_0 + \gamma_2}. \tag{8}$$

The expressions (5) for Q_3 and (6) for P_3 can be rearranged to give

$$\frac{\gamma_1}{2}(Q_3 - 1) = \frac{\gamma_1[-a\gamma_2 + ic\gamma_1\tan\gamma_1 l]}{\gamma_0 + a\gamma_2 - i[b(\gamma_0\gamma_2/\gamma_1) + c\gamma_1]\tan\gamma_1 l} \tag{9}$$

$$\frac{k_1^2}{2\gamma_1}(P_3 + 1) = \frac{k_1^2\gamma_0[1 - i(\gamma_2/\gamma_1)\tan\gamma_1 l]}{\gamma_1[\gamma_0 + \gamma_2 - i(\gamma_0\gamma_2/\gamma_1 + \gamma_1)\tan\gamma_1 l]} \tag{10}$$

where

$$a = k_0^2/k_2^2 \qquad b = k_1^2/k_2^2 \qquad c = k_0^2/k_1^2. \tag{11}$$

If it is assumed that the substrate is electrically thin so that

$$|k_1 l|^2 \ll 3 \tag{12}$$

the approximation

$$\tan\gamma_1 l \sim \gamma_1 l \tag{13}$$

is appropriate. With (1) and (11), it follows that the leading term in (9) is

$$\frac{\gamma_1}{2}(Q_3 - 1) \sim \frac{ic\gamma_1^3 l}{\gamma_0 - ic\gamma_1^2 l}. \tag{14}$$

Note that this approximation implies that $|c\gamma_1^2 l| \gg |a\gamma_2|$ in the significant ranges of the variable of integration λ in the integrals in (2). With (11), this is equivalent to

$$|k_2 l| \gg 1. \tag{15}$$

For carrying out the integrations in (2), the following rearrangement of (14) is convenient:

$$\frac{\gamma_1}{2}(Q_3 - 1) \sim \frac{ik_0^2 l}{k_1^2}\left\{\gamma_1^3\left[\frac{1}{\gamma_0 - i\left(k_0^2\gamma_1^2 l/k_1^2\right)} - \frac{1}{\gamma_0}\right] + \frac{\gamma_1^3}{\gamma_0}\right\}. \tag{16}$$

When this formula is substituted in (2), it is clear that the integrands that contain the square bracket in (16) become large only when λ is at or near $\lambda \sim k_0$. Hence, since $|k_1^2| > k_0^2$, it follows that $|k_1^2| > \lambda^2$ in the range of significant contributions to the integral, so that $\gamma_1 = (k_1^2 - \lambda^2)^{1/2} \sim k_1$. With these approximations, (16) becomes

$$\frac{\gamma_1}{2}(Q_3 - 1) \sim ik_0^2 k_1 l \left[\frac{1}{\gamma_0 - ik_0^2 l} - \frac{1}{\gamma_0}\right] + \frac{ik_0^2 l}{k_1^2}\frac{\gamma_1^3}{\gamma_0}. \quad (17)$$

When a similar rearrangement and approximations are made in (10), the corresponding result is

$$\frac{k_1^2}{2\gamma_1}(P_3 + 1) \sim \frac{k_1^2(1 - ik_2 l)}{k_2}\frac{\gamma_0}{\gamma_1}. \quad (18)$$

With (14) and (18), (2) becomes

$$E_{1\rho}(\rho,\phi,0) \sim -\frac{\omega\mu_0}{4\pi k_1^2}\cos\phi\left[2F_{\rho 0}(\rho) + F_{\rho 1}(\rho)\right] \quad (19)$$

$$E_{1\phi}(\rho,\phi,0) \sim \frac{\omega\mu_0}{4\pi k_1^2}\sin\phi\left[2F_{\phi 0}(\rho) + F_{\phi 1}(\rho)\right] \quad (20)$$

where

$$F_{\rho 0}(\rho) = -2e^{ik_1\rho}\left(\frac{k_1}{\rho^2} + \frac{i}{\rho^3}\right) \quad (21)$$

$$F_{\phi 0}(\rho) = -e^{ik_1\rho}\left(\frac{ik_1^2}{\rho} - \frac{k_1}{\rho^2} - \frac{i}{\rho^3}\right) \quad (22)$$

and $F_{\rho 1}(\rho)$ and $F_{\phi 1}(\rho)$ are, respectively, the second integrals in (2). With (17) and (18), these reduce to

$$F_{\rho 1}(\rho) = F_{\rho 2}(\rho) + F_{\rho 3}(\rho) \qquad F_{\phi 1}(\rho) = F_{\phi 2}(\rho) + F_{\phi 3}(\rho) \quad (23)$$

where

$$\left.\begin{matrix} F_{\rho 2}(\rho) \\ F_{\phi 2}(\rho)\end{matrix}\right\} \sim \frac{ik_0^2 l}{k_1^2}\int_0^\infty \frac{\gamma_1^3}{\gamma_0}\left[J_0(\lambda\rho) \mp J_2(\lambda\rho)\right]\lambda\, d\lambda + \left\{\begin{matrix} G_{\rho 2}(\rho) \\ G_{\phi 2}(\rho)\end{matrix}\right\} \quad (24)$$

$$\left.\begin{matrix} F_{\rho 3}(\rho) \\ F_{\phi 3}(\rho)\end{matrix}\right\} \sim -\frac{k_1^2(1 - ik_2 l)}{k_2}\int_0^\infty \frac{\gamma_0}{\gamma_1}\left[J_0(\lambda\rho) \pm J_2(\lambda\rho)\right]\lambda\, d\lambda \quad (25)$$

and

$$\left.\begin{matrix} G_{\rho 2}(\rho) \\ G_{\phi 2}(\rho)\end{matrix}\right\} = ik_0^2 k_1 l\int_0^\infty \left[\frac{1}{\gamma_0 - ik_0^2 l} - \frac{1}{\gamma_0}\right] \cdot \left[J_0(\lambda\rho) \mp J_2(\lambda\rho)\right]\lambda\, d\lambda. \quad (26)$$

The integrals in (24) and (25) can be expanded as follows:

$$F_{\rho 2}(\rho) = \frac{ik_0^2 l}{k_1^2}\left[k_1^2 I_1 - I_3\right] + ik_0^2 k_1 l I_5 \quad (27)$$

$$F_{\phi 2}(\rho) = \frac{ik_0^2 l}{k_1^2}\left[k_1^2 I_2 - I_4\right] + ik_0^2 k_1 l I_6 \quad (28)$$

$$F_{\rho 3}(\rho) = -\frac{k_1^2(1 - ik_2 l)}{k_2} I_7 \quad (29)$$

$$F_{\phi 3}(\rho) = -\frac{k_1^2(1 - ik_2 l)}{k_2} I_8 \quad (30)$$

where

$$\begin{aligned} I_1 &= \int_0^\infty \frac{\gamma_1}{\gamma_0}\left[J_0(\lambda\rho) - J_2(\lambda\rho)\right]\lambda\, d\lambda \\ &= 2i\left[-\frac{1}{k_1\rho^3} + \left(\frac{1}{\rho^2} + \frac{i}{2k_1\rho^3}\right)e^{ik_1\rho}\right] \\ &\quad - \frac{2k_1}{k_0}\left[\frac{1}{\rho^2} + \left(\frac{ik_0}{\rho} - \frac{1}{\rho^2}\right)e^{ik_0\rho}\right] \end{aligned} \quad (31)$$

$$\begin{aligned} I_2 &= \int_0^\infty \frac{\gamma_1}{\gamma_0}\left[J_0(\lambda\rho) + J_2(\lambda\rho)\right]\lambda\, d\lambda \\ &= \frac{i}{k_1\rho^3}(1 - 2ie^{ik_1\rho}) + \frac{2k_1}{k_0\rho^2}(1 - e^{ik_0\rho}) \end{aligned} \quad (32)$$

$$\begin{aligned} I_3 &= \int_0^\infty \frac{\gamma_1}{\gamma_0}\left[J_0(\lambda\rho) - J_2(\lambda\rho)\right]\lambda^3\, d\lambda \\ &= 2k_1^2\left[i\left(\frac{1}{\rho^2} + \frac{7i}{2k_1\rho^3}\right)e^{ik_1\rho} \right. \\ &\quad \left. - \frac{k_0}{k_1}\left(\frac{ik_0}{\rho} - \frac{2}{\rho^2} - \frac{2i}{k_0\rho^3}\right)e^{ik_0\rho}\right] \end{aligned} \quad (33)$$

$$\begin{aligned} I_4 &= \int_0^\infty \frac{\gamma_1}{\gamma_0}\left[J_0(\lambda\rho) + J_2(\lambda\rho)\right]\lambda^3\, d\lambda \\ &= 2k_1^2\left[\left(\frac{1}{k_1\rho^3} + \frac{3i}{2k_1^2\rho^4}\right)e^{ik_1\rho} - \frac{k_0}{k_1}\left(\frac{1}{\rho^2} + \frac{i}{k_0\rho^3}\right)e^{ik_0\rho}\right] \end{aligned} \quad (34)$$

$$\begin{aligned} I_5 &= \int_0^\infty \left[\frac{1}{\gamma_0 - ik_0^2 l} - \frac{1}{\gamma_0}\right]\left[J_0(\lambda\rho) - J_2(\lambda\rho)\right]\lambda\, d\lambda \\ &= -4ik_0^2 l e^{ik_0\rho}\left(\frac{\pi}{k_0\rho}\right)^{1/2} e^{-ip'}\mathscr{F}(p') \end{aligned} \quad (35)$$

$$\begin{aligned} I_6 &= \int_0^\infty \left[\frac{1}{\gamma_0 - ik_0^2 l} - \frac{1}{\gamma_0}\right]\left[J_0(\lambda\rho) + J_2(\lambda\rho)\right]\lambda\, d\lambda \\ &= -4k_0 l e^{ik_0\rho}\frac{1}{\rho}\left(\frac{\pi}{k_0\rho}\right)^{1/2} e^{-ip'}\mathscr{F}(p') \end{aligned} \quad (36)$$

$$I_7=\int_0^\infty \frac{\gamma_0}{\gamma_1}[J_0(\lambda\rho)+J_2(\lambda\rho)]\lambda\,d\lambda$$

$$=-2i\left(\frac{1}{\rho^2}+\frac{i}{2k_1\rho^3}\right)e^{ik_1\rho}+\frac{2k_0}{k_1}\left(\frac{1}{\rho^2}+\frac{i}{k_0\rho^3}e^{ik_0\rho}\right) \quad (37)$$

$$I_8=\int_0^\infty \frac{\gamma_0}{\gamma_1}[J_0(\lambda\rho)-J_2(\lambda\rho)]\lambda\,d\lambda$$

$$=-2i\left(\frac{ik_1}{\rho}-\frac{3}{2\rho^2}-\frac{5i}{8k_1\rho^3}\right)e^{ik_1\rho}-\frac{2k_0}{k_1\rho^2}$$

$$-\frac{2k_0}{k_1}\left(\frac{1}{\rho^2}+\frac{2i}{k_0\rho^3}\right)e^{ik_0\rho}. \quad (38)$$

In (35) and (36),

$$p'=k_0\rho\left(k_0^2l^2/2\right)$$

$$\mathscr{F}(p')=\frac{1}{2}(1+i)-\int_0^{p'}\frac{e^{it}}{(2\pi t)^{1/2}}\,dt. \quad (39)$$

The evaluation of these integrals follows the methods used in similar integrals in earlier papers [17]–[20]. Since the details require much space, they are omitted.

When (27)–(30) are used in (23) to obtain $F_{\rho 1}(\rho)$ and $F_{\phi 1}(\rho)$, these become

$$F_{\rho 1}(\rho)=ik_1l\left[\frac{k_0^2}{k_1}I_1-\frac{k_0^2}{k_1^3}I_3+k_1I_7+k_0^2I_5\right]-\frac{k_1^2}{k_2}I_7 \quad (40)$$

$$F_{\phi 1}(\rho)=ik_1l\left[\frac{k_0^2}{k_1}I_2-\frac{k_0^2}{k_1^3}I_4+k_1I_8+k_0^2I_6\right]-\frac{k_1^2}{k_2}I_8. \quad (41)$$

The appropriate integrals from (31)–(38) are now substituted in (40) and (41) and small terms of the order of k_0^2/k_1^2 and k_1/k_2 are neglected. The Fresnel-integral terms are also omitted since they are significant only at large distances. The results are

$$F_{\rho 1}(\rho)\sim-2ik_1l\left[\left(\frac{ik_0^2}{\rho}-\frac{k_0}{\rho^2}-\frac{i}{\rho^3}\right)e^{ik_0\rho}+i\left(\frac{k_1}{\rho^2}+\frac{i}{2\rho^3}\right)e^{ik_1\rho}\right] \quad (42)$$

$$F_{\phi 1}(\rho)\sim-2ik_1l\left[2\left(\frac{k_0}{\rho^2}+\frac{i}{\rho^3}\right)e^{ik_0\rho}+i\left(\frac{ik_1^2}{\rho}-\frac{3k_1}{2\rho^2}-\frac{5i}{8\rho^3}\right)e^{ik_1\rho}\right]. \quad (43)$$

These formulas can be substituted in (19) and (20) together with (21) and (22) to obtain

$$E_{1\rho}(\rho,\phi,0)\sim\frac{\omega\mu_0}{2\pi k_1^2}\cos\phi\left\{e^{ik_1\rho}\left[2\left(\frac{k_1}{\rho^2}+\frac{i}{\rho^3}\right)-k_1l\left(\frac{k_1}{\rho^2}+\frac{i}{2\rho^3}\right)\right]+ik_1le^{ik_0\rho}\left(\frac{ik_0^2}{\rho}-\frac{k_0}{\rho^2}-\frac{i}{\rho^3}\right)\right\} \quad (44)$$

$$E_{1\phi}(\rho,\phi,0)\sim-\frac{\omega\mu_0}{2\pi k_1^2}\sin\phi\left\{e^{ik_1\rho}\left[\left(\frac{ik_1^2}{\rho}-\frac{k_1}{\rho^2}-\frac{i}{\rho^3}\right)-k_1l\left(\frac{ik_1^2}{\rho}-\frac{3k_1}{2\rho^2}-\frac{5i}{8\rho^3}\right)\right]+2ik_1le^{ik_0\rho}\left(\frac{k_0}{\rho^2}+\frac{i}{\rho^3}\right)\right\}. \quad (45)$$

The corresponding expression for $E_{1z}(\rho,\phi,z)$ is readily derived from (3) in the form

$$E_{1z}(\rho,\phi,z)=\frac{i\omega\mu_0}{4\pi k_1^2}\cos\phi\left[2F_{z0}(\rho,z)+F_{z1}(\rho,z)\right] \quad (46)$$

where

$$F_{z0}(\rho,z)=-e^{ik_1r}\left(\frac{z}{\rho}\right)\left(\frac{k_1^2}{\rho}+\frac{3ik_1}{\rho^2}-\frac{3}{\rho^3}\right) \quad (47)$$

$$F_{z1}(\rho,z)=\int_0^\infty(Q_3-1)J_1(\lambda\rho)e^{i\gamma_1z}\lambda^2\,d\lambda. \quad (48)$$

With (17),

$$Q_3-1\sim2ik_0^2l\left[\frac{1}{\gamma_0-ik_0^2l}-\frac{1}{\gamma_0}\right]+\frac{2ik_0^2l}{k_1^2}\frac{\gamma_1^2}{\gamma_0}. \quad (49)$$

It follows that

$$F_{z1}(\rho,z)\sim2ik_0^2le^{ik_1z}\int_0^\infty\left[\frac{1}{\gamma_0-ik_0^2l}-\frac{1}{\gamma_0}\right]J_1(\lambda\rho)\lambda^2\,d\lambda+2ik_0^2le^{ik_1z}\int_0^\infty\frac{1}{\gamma_0}J_1(\lambda\rho)\lambda^2\,d\lambda-\frac{2ik_0^2l}{k_1^2}e^{ik_1z}\int_0^\infty\frac{1}{\gamma_0}J_1(\lambda\rho)\lambda^4\,d\lambda. \quad (50)$$

The three integrals are

$$\vartheta_1 = \int_0^\infty \left[\frac{1}{\gamma_0 - ik_0^2 l} - \frac{1}{\gamma_0} \right] J_1(\lambda\rho)\lambda^2 \, d\lambda = -2k_0^3 l e^{ik_0\rho} \left(\frac{\pi}{k_0\rho} \right)^{1/2} e^{-ip'} \mathscr{F}(p') \tag{51}$$

$$\vartheta_2 = \int_0^\infty \frac{1}{\gamma_0} J_1(\lambda\rho)\lambda^2 \, d\lambda = -e^{ik_0\rho} \left(\frac{k_0}{\rho} + \frac{i}{\rho^2} \right) \tag{52}$$

$$\vartheta_3 = \int_0^\infty \frac{1}{\gamma_0} J_1(\lambda\rho)\lambda^4 \, d\lambda = k_0^2 e^{ik_0\rho} \left(\frac{k_0}{\rho} + \frac{2i}{\rho^2} + \frac{3}{k_0\rho^3} \right) \tag{53}$$

where p' and $\mathscr{F}(p')$ are defined in (39). As with the integrals (31)–(38), the details of integration are omitted.

When these values are inserted in $F_{z1}(\rho, z)$, it turns out that the contributions from the first and third integrals are negligible because of the small factors k_0^2/k_1^2 and $k_0^2 l^2$, so that

$$F_{z1}(\rho, z) \sim -2ik_0^2 l \left(\frac{k_0}{\rho} + \frac{i}{\rho^2} \right) e^{ik_0\rho} e^{ik_1 z} \tag{54}$$

and

$$E_{1z}(\rho, \phi, z) \sim -\frac{i\omega\mu_0}{2\pi k_1} \cos\phi \left[\left(\frac{z}{\rho} \right) \left(\frac{k_1}{\rho} + \frac{3i}{\rho^2} - \frac{3}{k_1\rho^3} \right) e^{ik_1 r} + ik_1 l \left(\frac{k_0^2}{k_1^2} \right) \left(\frac{k_0}{\rho} + \frac{i}{\rho^2} \right) e^{ik_0\rho} e^{ik_1 z} \right] \tag{55}$$

where $r = (\rho^2 + z^2)^{1/2}$.

These are the final formulas for the tangential electric field along the air–substrate boundary in microstrip and the vertical electric field in its interior when excited by a unit horizontal electric dipole at $\rho = 0$ on the boundary with $d = 0$. They are good approximations when

$$k_0^2 < |k_1^2| \ll |k_2^2| \qquad |k_1 l| < 1 < |k_2 l|. \tag{56}$$

III. Comparison with the Two-Layered Region

It is instructive to compare the formulas for the electric field on the air–dielectric surface when the dielectric is a thin layer on a conducting half-space and when the dielectric is infinitely thick. The general formulas for the two half-spaces consisting of air (region 0) and a dielectric (region 1) when both the point of observation and the source are on the boundary $z = 0$ are readily obtained from [16]. They are

$$E_{1\rho}(\rho, \phi, 0) = \frac{\omega\mu_0}{2\pi k_1^2} \cos\phi \left[\left(\frac{k_1}{\rho^2} + \frac{i}{\rho^3} \right) e^{ik_1\rho} - \left(\frac{ik_0^2}{\rho} - \frac{k_0}{\rho^2} - \frac{i}{\rho^3} \right) e^{ik_0\rho} \right] \tag{57}$$

$$E_{1\phi}(\rho, \phi, 0) = -\frac{\omega\mu_0}{2\pi k_1^2} \sin\phi \left[\left(\frac{k_1}{\rho^2} + \frac{i}{\rho^3} \right) e^{ik_1\rho} - 2\left(\frac{k_0}{\rho^2} + \frac{i}{\rho^3} \right) e^{ik_0\rho} \right] \tag{58}$$

$$E_{1z}(\rho, \phi, z) \sim -\frac{i\omega\mu_0}{2\pi k_1} \cos\phi \left\{ \left(\frac{z}{\rho} \right) \left(\frac{k_1}{\rho} + \frac{3i}{\rho^2} - \frac{3}{k_1\rho^3} \right) e^{ik_1 r} + \frac{k_0^2}{k_1^2} \left[\left(\frac{1}{\rho^2} + \frac{3i}{2k_1\rho^3} \right) e^{ik_1 r} - \left(\frac{k_0}{\rho} + \frac{i}{\rho^2} \right) e^{ik_0\rho} e^{ik_1 z} \right] \right\}. \tag{59}$$

For simplicity, the Fresnel-integral terms have been omitted in (57)–(59) since they contribute negligibly when $k_0\rho \leqslant |k_1^2/k_0^2|$. In these formulas the terms multiplied by $e^{ik_0\rho}$ are the lateral wave that travels along the surface in the air, the terms multiplied by $e^{ik_1\rho}$ are the wave that travels in the dielectric.

The leading terms in the presence of a conducting plane below a dielectric layer of small thickness l are given in (44), (45), and (55). A comparison with (57)–(59) shows that the lateral-wave terms are the same except that, for the three-layered region, they are multiplied by the additional factor $-ik_1 l$. This factor appears because the field from the dipole source not only travels upward into the air and then horizontally along the boundary, but also travels downward through the dielectric substrate to be reflected by the conducting half-space. This reflected field travels back up through the dielectric, and is partly reflected, partly transmitted at the air boundary. The total field that enters the air and travels along the boundary has been multiply reflected and necessarily depends on l. The wave traveling in the dielectric includes terms that represent a direct transmission from the source to the point of observation and are independent of l and other terms that are modified by multiple reflections so that they depend on l.

IV. Properties of the Field

The field given by (44), (45), and (55) is very different from the surface wave described by Attwood [15]. Instead of a single wave that travels along the boundary partly in each region with a common phase velocity with a magnitude between c and $c\epsilon^{1/2}$, there are two waves—the lateral wave that travels with the velocity c in the air and a wave in the dielectric that travels with the velocity $c\epsilon^{1/2}$. In this respect the field along the air–dielectric boundary in the three-layered microstrip is very much like the field along the air–dielectric boundary of two half-spaces. This field

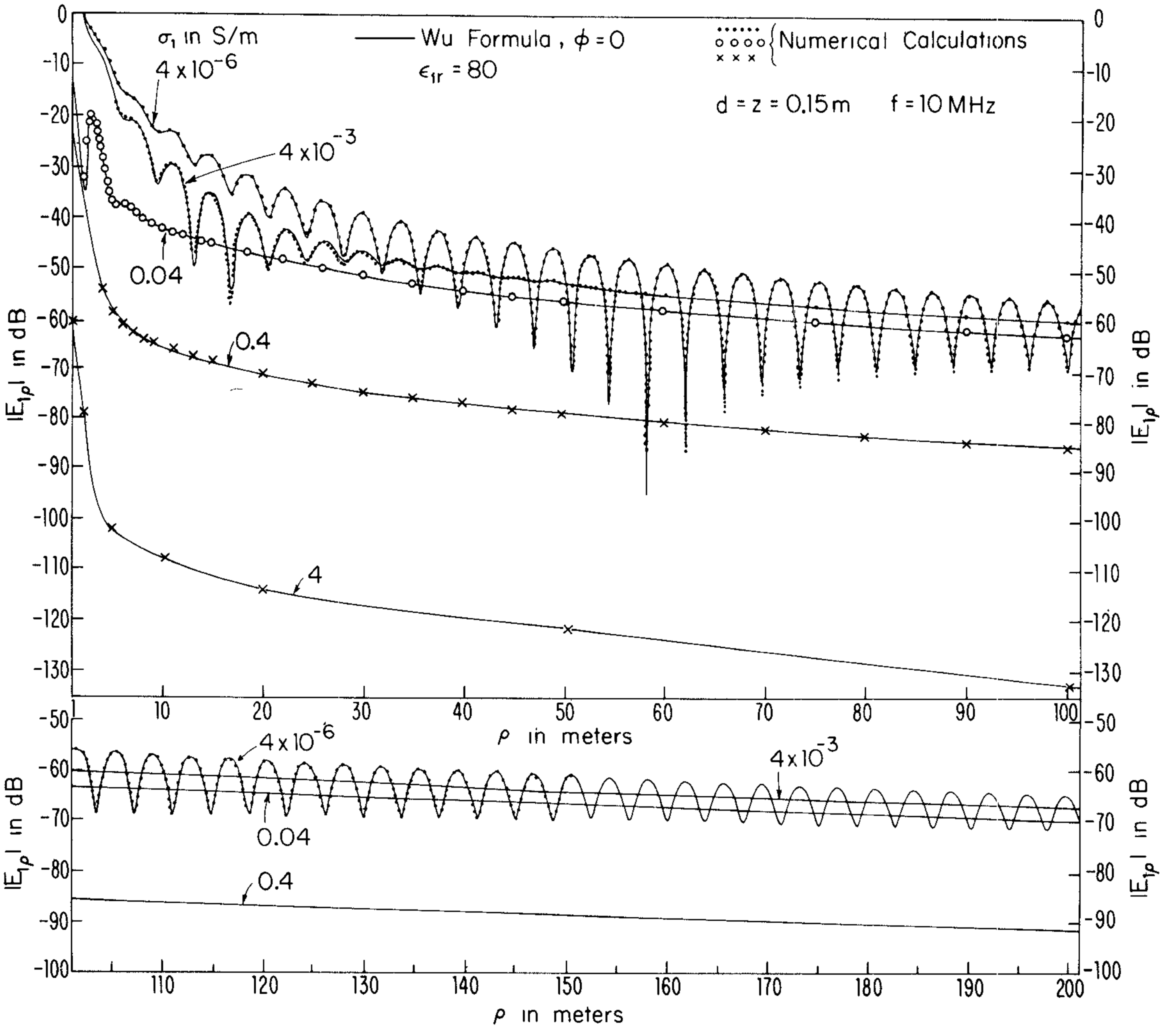

Fig. 2. $|E_\rho|$ at depth z of horizontal electric dipole at depth d in water below air.

has been studied in detail for E_ρ and the interference effects generated by the two outward-traveling waves with different phase velocities have been verified [17], as shown in Fig. 2 for the boundary between air and water with a range of conductivities. The interference patterns along a good dielectric such as distilled water are seen to persist for large distances along the boundary.

Note that when a dipole operating at a single fixed frequency generates two waves that travel outward with different phase velocities, the effect is similar to, but is not, dispersion. In the case at hand, the permittivities of air and the substrate are assumed to be constants independent of the frequency so that the phase velocities associated with them are also frequency-independent. However, a signal that propagates without dispersion at two different phase velocities is distorted in transmission in a manner similar to that observed with dispersion.

The interaction between currents in conductors that are close together on the air–dielectric boundary is determined by the quasi-static components that dominate when

$$k_0\rho < |k_1\rho| < 1. \tag{60}$$

They are

$$E_{1\rho}(\rho,\phi,0) \sim \frac{\omega\mu_0}{2\pi k_1^2\rho^3}\cos\phi\left[k_1 l\left(1-\frac{1}{2}i\right)+2i\right] \tag{61}$$

$$E_{1\phi}(\rho,\phi,0) \sim \frac{\omega\mu_0}{2\pi k_1^2\rho^3}\sin\phi\left[k_1 l\left(2+\frac{3}{8}i\right)+i\right]. \tag{62}$$

Note that $E_{1\rho}(\rho,0,0)$ and $E_{1\phi}(\rho,\pi/2,0)$ are comparable in magnitude. This means that collinear coupling is as strong as broadside coupling—a situation very different from that in a homogeneous medium.

At larger distances defined by

$$1 \leqslant k_0\rho \leqslant |k_1^2/2k_0^2| \qquad |k_1^2\rho^2| > 1 \tag{63}$$

the leading terms in the field are

$$E_{1\rho}(\rho,\phi,0) \sim -\frac{\omega\mu_0 k_0^2 l}{2\pi k_1}\frac{e^{ik_0\rho}}{\rho}\cos\phi, \tag{64}$$

$$E_{1\phi}(\rho,\phi,0) \sim \frac{i\omega\mu_0}{2\pi}\frac{e^{ik_1\rho}}{\rho}(k_1 l-1)\sin\phi. \tag{65}$$

Note that $E_{1\phi}(\rho,\pi/2,0)$ is now greater than $E_{1\rho}(\rho,0,0)$.

V. Conclusions

The electric field generated by a horizontal electric dipole along the boundary between the air and the substrate in microstrip is complicated. However, the integrated formulas (44), (45), and (55) are both simpler and physically more readily understood than the integrals (2) and (3) which they approximate. Along the boundary, $E_\rho(\rho,\phi,0)$ and $E_\phi(\rho,\phi,0)$ each consists of two parts: a lateral wave characterized by the exponential factor $e^{ik_0\rho}$ that travels in the air and a direct wave in the substrate characterized by the exponential factor $e^{ik_1\rho}$. These two components determine the interaction between currents in conductors located on the air–substrate boundary. $E_{1z}(\rho,\phi,z)$ in the substrate also includes a lateral wave that travels along the surface and then down into the substrate and a direct wave that travels directly through the substrate. It determines the coupling between vertical currents in connections from the air surface to the conducting plane.

The distortion of signals as they propagate along the surface of microstrip is not due to dispersion in the ordinary sense that the permittivities of the air or the substrate are frequency-dependent; rather it is due to the fact that each signal separates into two parts, the one traveling in the air, the other in the dielectric with different phase velocities. If the permittivity of the substrate is frequency-dependent, the part of the signal traveling in it will suffer dispersion; the lateral-wave part in the air will not.

The fields determined in this paper are those of an infinitesimal electric dipole on the air–substrate boundary. The extension of the new theory to microstrip transmission lines and antennas offers the possibility of challenging new insights into their properties, notably dispersion and coupling.

Acknowledgment

The author wishes to thank B. H. Sandler for carrying out the numerical calculations and M. Owens for her help in correcting the manuscript and preparing it for publication.

References

[1] T. T. Wu, "Theory of the microstrip," *J. Appl. Phys.*, vol. 28, pp. 299–301, Mar. 1957.
[2] C. Shafer, "Dominant mode of the microstrip transmission line," Cruft Laboratory Tech. Rep. No. 257, Harvard University, Cambridge, MA, Nov. 1957.
[3] R. H. Jansen, "The spectral domain approach for microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1043–1056, Oct. 1985.
[4] E. J. Denlinger, "A frequency-dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.
[5] Y. Fujiki, Y. Hayashi, and M. Suzuki, "Analysis of strip transmission lines by iteration method," *Electron. Commun. Japan*, vol. 55, pp. 74–80, 1972.
[6] G. Kowalski and R. Pregla, "Dispersion characteristics of single and coupled microstrips," *Arch. Elek. Übertragung.*, vol. 26, pp. 276–280, 1972.
[7] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, July 1973.
[8] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 691–694, July 1979.
[9] M. Hashimoto, "A rigorous solution for dispersive microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1131–1137, Nov. 1985.
[10] S. Deibele and J. B. Beyer, "Measurements of microstrip effective relative permittivities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 535–538, May 1987.
[11] W. J. Getsinger, "Microstrip dispersion model," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 34–39, Jan. 1973.
[12] R. L. Veghte and C. A. Balanis, "Dispersion of transient signals in microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1427–1436, Dec. 1986.
[13] K. R. Carver and J. W. Mink, "Microstrip antenna technology," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 2–24, Jan. 1981.
[14] R. J. Mailloux, J. F. McIlvenna, and N. P. Kernweis, "Microstrip array technology," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 25–37, Jan. 1981.
[15] S. S. Attwood, "Surface-wave propagation over a coated plane conductor," *J. Appl. Phys.*, vol. 22, pp. 504–509, Apr. 1951.
[16] R. W. P. King, "Electromagnetic surface waves: New formulas and applications," *IEEE Trans. Antennas Propagat.*, vol. AP-33, pp. 1204–1212, Nov. 1985.
[17] T. T. Wu and R. W. P. King, "Lateral waves: A new formula and interference patterns," *Radio Sci.*, vol. 17, pp. 521–531, May–June 1982.
[18] ——, "Lateral waves: New formulas for $E_{1\phi}$ and E_{1z}," *Radio Sci.*, vol. 17, pp. 532–538, May–June 1982; Correction, vol. 19, p. 1422, Sept.–Oct. 1984.
[19] R. W. P. King and T. T. Wu, "Lateral waves: Formulas for the magnetic field," *J. Appl. Phys.*, vol. 54, pp. 507–514, Feb. 1983; Erratum, vol. 56, p. 3365, Dec. 1984.
[20] R. W. P. King, "New formulas for the electromagnetic field of a vertical electric dipole in a dielectric or conducting half-space near its horizontal interface," *J. Appl. Phys.*, vol. 53, pp. 8476–8482, Dec. 1982; Erratum, vol. 56, p. 3366, Dec. 1984.

Ronold W. P. King (A'30–SM'43–F'53–LF'71) was born in Williamstown, MA, on September 19, 1905. He received the B.A. and M.S. degrees in physics from the University of Rochester, Rochester, NY, in 1927 and 1929, respectively, and the Ph.D. degree from the University of Wisconsin, Madison, in 1932, after having done graduate work at the University of Munich, Munich, Germany, and at Cornell University, Ithaca, NY.

He served as a Teaching and Research Assistant at the University of Wisconsin from 1932 to 1934, and as an Instructor and Assistant Professor of Physics at Lafayette College, Easton, PA, from 1934 to 1937. During the academic year 1937–1938 he was a Guggenheim Fellow in Germany. In 1938 he joined the faculty of Harvard University, Cambridge, MA.

Dr. King has been Gordon McKay Professor of Applied Physics at Harvard from 1946 to 1972, when he became Gordon McKay Professor of Applied Physics, Emeritus. He was again a Guggenheim Fellow in 1958. He is a fellow of the American Physical Society and the American Academy of Arts and Sciences, a corresponding member of the Bavarian Academy of Sciences, and a member of the American Association for the Advancement of Science, Commission B of the International Scientific Radio Union, Phi Beta Kappa, and Sigma Xi. He is the author or coauthor of ten books, numerous articles in books and encyclopedias, and over 250 original papers in technical journals.